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FINAL REPORT

OF

RESEARCH

ACCOMPLISHMENTS

UNDER ONR N00014-85-K-0150

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Submitted by

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Introduction: In the area of signal analysis, one increasingly encounters situations in which the classical second order theory fails. Either the observed signals/noise are not second—order or they are observed on a lattice which has no specific direction. In the first case, there is no spectral analysis available and in the second the analytic results obvious in the classical theory are not easily available. Professors LePage and Mandrekar have developed mathematical techniques, based on the Theory of Errors, to create alternative approaches to the spectral and spatial analysis of such signals.

Under ONR N00014— 85—K—0150 Professors LePage and Mandrekar, together with their students J. Kinateder (ONR grant), K. Kinateder (P. R. Harris fellowship), Z. Liu (ONR grant), A. Taraporevala (ONR grant), B. Thelen (ONR grant and IBM Watson fellowship), and S. Zhang (ONR grant), have developed important new mathematical methods for dealing with non— Gaussian noise, particularly symmetric α—stable (SαS) noise. These new methods, based on the Theory of Errors, have laid the foundation for developing alternative methods for spectral and spatial analysis of signals and noise in non—Gaussian environments.

The core of the research accomplishments which have already been made by Dr. LePage and Dr. Mandrekar under ONR N00014— 85—K—0150 will be described below together with references and a brief discussion of their perceived importance. One measure of the success of this work is the large and growing number of research publications which have already made fundamental use of them.

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RESEARCH ACCOMPLISHMENTS UNDER ONR N00014–85–K–0150

Research Done By Dr. LePage
Under The Support Of Grant ONR N00014–85–K–0150

A. **Series constructions of stable random processes and invariance principles:** Dr. LePage [23, appendix] has developed series constructions of stable processes (and more generally operator stable processes) now generally known as the LePage Representation. This construction is a widely used tool in the study of stable processes and has been used to obtain new results ranging from path properties (M. Marcus, G. Pisier), extrema of stable processes (G. Samorodnitsky), Bootstrap (K. Knight) [7], stable measures of balls (M. Lewandowski, et. al.) and a host of other applications (Ledoux, Tallagrand). Recently Dr. LePage [6] generalized and simplified the series construction of stable processes by proving that it follows from the method of marking of a Poisson process.

Another fundamental use of series constructions is made by Dr. John Kinateder in his PhD thesis An Invariance Principle Applicable to the Bootstrap [9], written under the direction of Dr. LePage and supported by the grant. In the thesis Dr. Kinateder establishes a series construction of the LePage–type for sums of i.i.d. random variables in the domain of attraction of a SαS distribution and proves a new type of invariance principle describing the convergence of such constructions to the LePage series for the limit laws. This invariance principle is applicable to describing the joint behavior of the sample sums, re–sampled sums, and order statistics, and will provide the basis for entirely new conditional inference approach to statistical analysis in stable noise environments (see Seamless Resampling below).

B. **Gaussian Slicing:** Dr. LePage [23, appendix] proved that every symmetric α–stable (SαS) random vector or process, irrespective of time domain or range, has the distribution

of a mixture of Gaussian distributions. That is, the probability space may be sliced into (infinitely many) slices on each of which the stable process is conditionally a Gaussian process (dependent in a particular way upon that slice). Dr. LePage [6] has recently extended this to the underlying stable Poisson point process (see Point Process Approach to Stable Processes below).

Using Gaussian slicing, powerful methods available from Gaussian theory may be brought to bear on SαS processes with sometimes surprising results. Dr. LePage [3] [4] exploited Gaussian slicing to establish $E(X^2(t) \mid X(t-\delta), X(t-2\delta)) < \infty$ a.s. for the Fourier transforms $X(t) = \int e^{it \cdot \lambda} Z(d\lambda)$ of complex (conjugate) symmetric independent increments stable noise, even though $E X^2(t) = \infty$ for every $\alpha < 2$. This result, the first of its type, means that second order prediction is valid for such processes and has precipitated a number of papers (Cambanis, Taqqu). Other researchers have applied slicing to Slepian type inequalities [G. Samorodnitsky), and most recently to the stable measures of balls (M. Lewandowski, et. al). The latter actually obtains the Gaussian result also by these methods.

Aside from such technical uses of slicing it has deep implications for prediction and estimation in SαS environments. In an invited talk before the Conference on Stable Measures and Extremes held in conjunction with the regional IMS Boston Meeting (1987), Dr. LePage gave an invited talk showing that prediction for the SαS case is, by Gaussian slicing, a Bayes problem with the data consisting of a single realization in which one observes the path of a Gaussian process whose covariance operator is itself a random $\alpha/2$ positive stable covariance process. It follows that prediction can be viewed in two steps: (a) use the data (and the $\alpha/2$ stable model!) to estimate a covariance operator (i.e. to estimate the slice) and then (b) use the best Gaussian predictor for that estimated slice. Actual simulations done on a computer show that for $\alpha = 1$ (Cauchy processes) the resulting estimator of location for i.i.d. data is virtually identical to the median! So the approach does appear to be correct. New results of Dr. LePage and Dr. Kinateder [7] show

how a Bootstrap–like resampling method can **automatically** calibrate the estimation procedure for the index of stability and bypass all the specialized calculations of the Bayes approach. These results will appear in the **Proceedings of the IMS Bootstrap Conference** recently held at MSU on May 15–16, 1990 (see **Seamless Resampling** below).

C. **Point Process Approach to Stable Processes**: Dr. LePage [6] recently proved that Gaussian slicing holds also for the stable Poisson point processes which underlie stable processes. That is, $\alpha < 2$ symmetric stable (with respect to set–union of points and scale change) Poisson point processes may be represented as mixtures of Gaussian point processes.

Furthermore, as also proved in [6], stable Poisson point processes exist for all indices of stability $\alpha \neq 0$ and have applications to e.g. density estimation and super–efficient estimation of location. Thus stable noise processes are identified with the summation operator applied to stable point processes, but if we look beyond sums to other functions of random point disturbances then all indices of stability $\alpha \neq 0$ become relevant and have varying applications. This perspective will probably come to dominate the study of stable processes.

D. **Seamless Resampling.** Dr. LePage and Dr. Kinateder [6] develop a bootstrap–like resampling method for the sample average of i.i.d. r.v. in the domain of attraction of a symmetric stable law of arbitrary index $\alpha \leq 2$ to which they give the name Seamless Resampling. It recovers the limiting conditional distribution of the sum of the errors, conditional on the vector of absolute values of these same errors. This is an improvement on what was earlier presumed to be the goal: recovery of the unconditional limit law of the sum of the errors and leads to narrower confidence intervals. The conditional inference possible by seamless resampling is not present in the $\alpha = 2$ case and is an entirely new aspect of inference for SαS discovered under ONR N00014–85–K–0150.

With the seamless resampling procedure (described in more detail elsewhere in this proposal) it is possible to use resampling to **automatically** identify the limit model and

adjust for the correct values of the unknown nuisance parameters without explicitly incorporating any such information in the description of the statistical method itself.

Contrast these results with what was previously known. Athreya proved that ordinary Bootstrap (not seamless resampling!) fails for the case of the sample average in this same case if $\alpha < 2$. Giné and Zinn proved that in a certain sense Bootstrap requires finiteness of the second moment. All of this makes the seamless resampling result extremely interesting since one of the difficulties of developing statistical methods capable of dealing with stable noise has been to work out the needed distribution theory. With resampling methods it may be possible to bypass this step altogether.

E. **Optimality methods based on expected logarithm.** Entropy methods such as the Burg algorithm used successfully in spectral density estimation often have many useful applications but little theory behind them. To a large degree it is not known why they work. Dr. LePage has been studying the connections between entropy maximization or minimization and the growth of random products. This has connections with the work of L. Breiman (4th Berekely) on the Kelly Principle and T. Cover on portfolio optimization. In such problems the idea is to control a random process so that it grows toward infinity most rapidly, or decays toward zero least rapidly.

Dr. LePage and Dr. Schreiber [5] connected this idea with sequential estimation. They used martingale decomposition methods to establish rapid growth of a partial product of random variables indexed by parameters when these parameters are at each stage chosen to maximize the conditional expectation of the logarithm of the next term of the product given the infomation then available. They applied this result to sequentially estimating a parameter in such a way as to make the product of the likelihoods decrease least rapidly.

Dr. LePage [12] addressed a similar problem for particular diffusions using the Ito calculus to develop stronger forms of the rapid growth result. The basic conclusion is that under appropriate conditions the drift term in the Ito differential of the log of the process should be maximimized in order that the process grow most rapidly to infinity.

Dr. LePage [11] considered random products plotted at random times and proved that under some conditions the product tends most rapidly to infinity in the random time scale if at each stage one maximizes the ratio of the conditional expected log of the next term of the product to the conditional expected time to obtain it. This result is being studied for its potential application to real–time estimation and control.

Reports Supported Under

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Research Done By Dr. Mandrekar

Under The Support Of Grant ONR N00014–85–K–0150

The research which was proposed under the grant has several components; Linear prediction of SαS processes, Multiparameter processes, robust detection, and higher order spectra. In the linear prediction it was demonstrated by Dr. Mandrekar [4] that the time domain and spectral domain theories have to be developed separately under the then existing techniques. It was shown [4] that all existing work on prediction problem for strongly harmonizable SαS processes can be derived by adapting classical methods. However to handle general problem one needs Generalized spectrum. This was introduced in [4]. As isometries in Lp have no orthogonality properties except when p = 2, the definition of spectrum covered non–stationary processes thus giving a new analytic tool even in the case p = 2. In general,it is also shown [9] that SαS processes are images of L_2 processes. The existing orthogonality techniques used in the field were shown to be a special case of semi inner product of Lumer and the scope of these techniques in applications was expanded in ([12] [15]). The time domain analysis was connected to the geometry of linear subspaces generated by the process and this was used to study ARMA processes in exchangeable random variables ([8]). Dr. Mandrekar has been successful in computing the Generalized Spectrum for moving averages and thus the techniques for the development of relation between the time domain and the spectral domain for SαS processes are ready. To handle some nonlinear problems the structure of processes of special type (n–ple Markov) have been established [7]. These relate to the spherical averages of isotropic multidimensional SαS processes similar to Levy Brownian motion. Thus one can study prediction of spherical averages of multiparameter SαS processes. In addition, the basic analytic tools on Beurling Theorem, inner outer factorization and the structure of invariant subspaces of L^2 over a polydiscs have been established ([1], [5]) by Dr. Mandrekar. In addition, Drs. Kallianpur and Mandrekar established the time domain

analysis of multiparameter processes.

Techniques for the study of limit theorems for non–linear functionals of i.i.d. triangular arrays have been established starting with [3]. In the stable case, these give operator stable processes with diagonal operators i.e. multidimensional stable with components stable with different index. These techniques will enable one to study limit theorems for higher order spectra and those related to ARMA processes with noise in the domain of attraction of stable law. The study in ([11], [14]) is related to robust detection of signals with finite fisher information. In particular, this includes SαS product measures. The work in statistical analysis of Markov space–time processes have been initiated by Dr. Mandrekar [10] which is motivated from his study related to Markov type processes [2]. For this initial study was carried out for point process techniques in statistical analysis [6]. However, the general problem of space time processes seems to be open even for Markov equilibrium measures. This is being studied by Mr. Zhang [13] under the direction of Dr. Mandrekar.

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PROBABILISTS MEET IN WARSAW

Leading probability researchers assembled in Warsaw May 21 – June 1, 1990 at the invitation of the Stephen Banach Center of the Polish Academy of Science. Under discussion: the rapidly evolving field of stable random processes used to model noise and measurement errors that depart significantly from the normal [bell] curve of errors.

Introduction. One of the common sources of stable errors is a summation of many small equally distributed random disturbances. Stable errors thus retain their distributional form [are stable] when subjected to linear operations such as filtering. Normal errors are the most studied example of stable errors although it has been known for decades that the non–normal stable errors are better able to model more volatile or erratic noise. The techniques used currently for studying stable errors have only recently become available.

Raoul LePage, Michigan State University, developed three of these techniques. One [LePage Series Representation] breaks down every stable process into a series of progressively smaller terms connected with particular order statistics. Another [Gaussian Slicing] represents every symmetric stable process as a mixture of normal processes. The third and newest [Seamless Resampling] enables statistical estimates of the sampling error of linear statistics to be made automatically without explicit reference to the specific form of the symmetric stable noise present in the system.

The series construction is typically used to study the effects of various operations [on stable noise processes] through their action on the larger terms of the series. It has proven useful in understanding the oscillatory behavior and extremes of stable processes. Gaussian slicing is used to bring the methods of normal processes to bear on the problems of symmetric stable processes. Seamless resampling allows the development, just begun, of simple statistical procedures that automatically deal with stable noise, if it is present, and produce the usual results in the normal case.

It should be pointed out that many of the results developed for normal and

non—normal stable processes carry over to a far larger class of processes in what is termed the domain of attraction of stable.

Dr. LePage described the meetings as, " a part of an effort to understand the broad theoretical structure [of stable processes] so we can go on to engineering solutions." LePage says this particular conference was important since rapid developments were fragmenting the efforts of researchers.

An example illustrates some of the newer understandings achieved concerning stable processes. If stable noise enters a system through wave amplitudes, then an attempt to break the resulting composite wave form down into frequencies using [the usual] methods calibrated for normal noise will proceed without incident and will yield oscilloscope tracings which quickly lock—on as the experiment is continued over time. In the case of normal noise the oscilloscope tracing would not vary if the experiment were repeated [restarted]. However, for stable [but not normal] noise repeating the experiment will give an oscilloscope tracing which somehow resembles the first but is decidedly different.

Stable noise entering the system in other ways than through amplitudes will have different, but still anomalous effects.

Using methods calibrated for stable noise [methods that allow for extreme values in noise components] one can cut through such anomalies to discover the underlying structure.

Using an index of stability one can determine whether stable models are appropriate. Such methods may shortly be out of date. Automatic methods which bypass this step appear to be on the way. "Methods being developed now are sensitive to the possibility of stable noise," says LePage, "one can have procedures that adapt themselves to non—normal noise if it is present but otherwise behave conventionally."

Highlights of the conference. Rapid progress in the study of stable processes has resulted in the relative isolation of key groups working in different parts of the world.

Dr. V. N. Zolotarev, Moscow, U.S.S.R. showed work on the simultaneous

convergence of sums of powers of different orders. This problem has a solution in terms of the series representations.

Dr. V. Paulauskas, Vilnius, U.S.S.R. visited M. Ledoux, Strasbourg for the month preceding the conference. During the final week of his visit he and Dr. Ledoux developed a solution to the problem of rates of convergence in the central limit theorem for stable laws that is based on the series representation.

Contact between the Polish researchers and the U.S. is very good. Drs. M. Lewandowski, M. Ryznar, and T. Zak of Wrockaw showed a very nice result which uses the series construction to establish an Anderson–type inequality for the stable measure of spherical regions. They actually obtain the result for normal errors as a by–product of the general stable case. This result is a basic means by which one can show that random processes can be essentially regarded as identically zero. It is used to, for example, truncate a series construction after a fixed number of terms. It might also be used to judge how many terms of the series are sufficient to simulate the process to a given accuracy.

Dr. J. Rosinski of Tennessee showed a new series representation and with it some extensions of Gaussian slicing. Dr. B. Rajput, also of Tennessee, described the support of stable measures.

Dr. G. Samorodnitsky, Cornell used Gaussian slicing to prove a Slepian inequality [useful in signal detection] for stable processes.

Dr. Mandrekar, Michigan State University, showed a new generalized spectrum which applies in particular to every stationary stable process. The method is very general and, as applied to cases where Gaussian slicing is available, produces the spectrum obtained by that method. It also applies to ARMA systems driven by symmetric stable noise and virtually all known cases where a spectrum had earlier been obtained by differing methods. It is now important to study the applications of this spectrum to problems of prediction and estimation.

Dr. LePage showed a new bootstrap–like method for symmetric stable errors called

Seamless Resampling which is able to recover the conditional distribution of the sum of errors given the vector of the absolute values of these same errors. In effect, the method draws samples from the data in the course of which information is obtained about the absolute values of the errors present in the data. It is simple to use and automatically adjusts for stable noise when that is present. Seamless resampling is now under development for regression and time series applications where protection from non—normal noise is needed. It is called seamless because it adapts to non—normal errors just as well as to normal errors.

Dr. M. Maejima, Kyoto, Japan showed results characterizing self—similarity in particular classes of stable processes. Self—similarity is a property relating changes in the time scale to changes in the scale of the oscillations of the process.

Comments on the conference. The conference was very well organized and attended by over 30 internationally known experts. The Banach Center hosts, Dr. S. Kwapian and Dr. A. Weron, and staff gave generously of their time to look after the needs of the conferees. Many more researchers were present than can be highlighted here. Also, a number of computer scientists from Poland, E. Germany, U.S.S.R. came to the conference to learn about stable processes and means of simulating them for use in modeling. There seems to be consensus that the series construction has made such computer simulations of stable processes generally. feasible for the first time but that much more efficient methods doubtless exist and must be sought.

A new Steinhaus Center is starting in Wrockaw with contacts at several U.S. universities. Research on stable processes will also be high on the agenda for this group headed by Dr. A. Weron.